For Investment Geometric Problems

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Abstract. Solving geometric problems of special importance is the transformation of a plane called inversion. When solving geometric problems, the transformation of the plane called inversion is of special importance. An inversion is a mapping of a plane that a set of directions and a circle maps to the same set, and in doing so it can map a line either to a line or to a circle, and it can also map a circle to either a line or a circle.

Keywords: Inversion, Euler’s circle, Feuerbach’s theorem, Apollonius problem.

Introduction

Often, when solving geometric tasks, we use one of the transformations of the plane. In this article, we will get acquainted with inversion, symmetry with respect to the circle, a mapping which in many respects is analogous to symmetry with respect to the direction. Examples of inversion are not often found in nature, but as a relatively young method, it presents great relief in many geometric problems and even in the problems of mechanical nature.

In this article, the conclusions on geometric inversion are presented, to which all the mentioned mathematicians agreed upon, and at the end, discussions about its application through interesting tasks and its use in the real world are provided.

Properties of Inversion

In this section, we will study the significant properties of inversions. We will also consider cases, such as when the inversion maps the direction in the direction, when the direction is in the circle, when the circle is mapped in the direction, and when the circle is mapped to the circle. We have listed some specific inversion maps as follows.

THEOREM 1.

Let $p \subset M$ be the direction through the center $O$ of inversion $I_O$, then $I_O(p \setminus \{O\}) = p \setminus \{O\}$.

Proof.

The proof is a direct consequence of the fact that $O, T,$ and $T'$ must be collinear points and that the $I_O$ is a bijection.

THEOREM 2.

Let $A, A'$ and $B, B'$ pairs belong to the inverse points of $I_O$ with the circle of the invertebrate $k(O, R)$, then it is worth it.

\[ \angle OAB = \angle OB'A' \quad e., \quad \angle OBA = \angle OA'B' \]

Proof.

Let $A$ and $A'$ be associated points of inverse $I_D$, then $|OA| : |OA'| = R^2$ (Figure 1). Analogously, $|OB| : |OB'| = R^2$; however,

\[ |OA| : |OB| = |OB'| : |OA'| \Rightarrow |OA| : |OB'| = |OB| : |OA'|. \]

Figure 1. Pairs of associated inversion points.
Since $\angle AOB = \angle B'O'A'$ is followed by $\triangle OAB \sim \triangle O'B'A'$, from which we get $\angle OAB = \angle O'B'A'$ and $\angle OBA = \angle O'A'B'$.

The immediate consequence of the theorem is the following assertion.

**Corollary.**

The inversion is determined by the inversion center and the pair of associates.

**Theorem 3.**

The direction $p$ which does not pass by the center $O$ of inversion $I_O$ is mapped into a circle which passes through the center $O$ of inversion.

**Proof.**

Let $p \subset (M\setminus\{O\})$ be some direction (Figure 2). Make sure that you’re okay with the spu wall from $O$ to $p$, and $N' = I_O(N)$. Let $T \subset p$ be any of the directions $p$, and $T' = I_O(T)$.

![Figure 2](image)

**Figure 2.** The image of the direction that does not pass by the center of inversion is a circle.

**Theorem 2,** $\angle ON'T = \angle OT'N' = 90^\circ$. From this, according to the turn of Tales’ teaching, it follows that $T'$ is located on the circle $p'$ with $ON'$ the diameter. So, each of these directions, $p$, is mapped in that circle, $p'$. As soon as $I_O$ is a bounce, the claim is followed immediately.

**Remark.**

If the direction fires the inversion in the points $M$ and $N$, then the circle $p'$ is determined by the nonlinear points $O, M$, and $N$ (Figure 3); and if the direction $p$ enters the circle of inversion in point $D$, then $p'$ is a circle with a diameter $OD$ (Figure 4).

In order to demonstrate the following inversion properties, it is necessary to introduce the notion of potency in this respect in view of a crochet. First, we prove the following theorem.

![Figure 3](image)

**Figure 3.** A picture of the direction that does not pass through the center of inversion and shows a vertex of inversion.

**Theorem 4.**

Let $k(S, r)$ be a circle, $T$ be any plane, and $q$ be any direction that passes through $T$, and $A$ and $B$ be some points on the circle, then the product $p = |TA| \cdot |TB|$ does not depend on the choice of the direction $q$ through $T$, and we call it the potential $p$ point $T$ with respect to the circle $k$.

**Proof 1.**

Let $T$ be a point on the circle $k(S, r)$ (Figure 5), then $T = A$ or $T = B$. If $T = A$, then $|TA| = 0$; and if $T = B$, then $|TB| = 0$. From this, it follows that $|TA| \cdot |TB| = 0$.

**Proof 2.**

Let $T$ be a point within the circle $k(S, r)$. Let $q_1$ and $q_2$ be the directions passing through the point $T$, that is, $q_1 \cap k = \{A, B\}$ and $q_2 \cap k = \{C, D\}$ (Figure 6). Since the circumferential corners over the same arc are the same, they follow:

$$\angle CAB = \angle CDB;$$

$$\angle DCA = \angle DBA.$$
It follows that the triangles $TAC$ and $TDB$ are similar, and hence, it is valid
\[
\frac{|TA|}{|TD|} = \frac{|TC|}{|TB|},
\]
therefore, $|TA| \cdot |TB| = |TC| \cdot |TD|$.

\section*{Conclusion}

After all, it remains to us only to identify a couple of obvious things. First of all, geometric inversion in relation to the circle, with its many and very affected properties, has a very large application in non-standard geometric problems. When using invariants during the mapping itself, many difficult detectable dependencies between circles become apparent as addictions between the real ones. In addition, we also presented the application of inversion in real life, which was found in an attempt to turn the circular motion into a straight line motion.

\section*{Bibliography}


\section*{CITATION OF THE PAPER}

In this paper, after considering the properties of inversion, Feuerbach’s theorem and analytical proof of the theorem are given. Feuerbach’s theorem, one of the most beautiful theorems on the geometry of a triangle, states that the Euler circle of a triangle touches a triangle inscribed circle inside and all three triangles assigned a circle outside. The paper discusses the solution of the problem in an algebraic way and shows how this problem can be solved using inversion.

Therefore, inversion is used for various constructions related to circles so that by applying the inversion some of the circles are mapped in directions and thus the task is reduced to an analog simpler problem in which some circles are replaced by directions.